APPENDIX D - LOSSES AND ELONGATIONS

CALCULATIONS

The following appendix contains all the necessary information and formulas for calculating prestress losses and elongations for prestressed, post-tensioned structures. Included are example calculations for a simple-span structure stressed from one end and for a continuous structure stressed from one end. Also included is an anchor set example calculation.

It should be understood that the formulas and calculations are approximate and the engineer should apply reasonable tolerances when comparing the actual field measured elongations with those that are theoretical.

LOSSES

Post-tensioning of prestress box girder bridges must consider stress losses that will occur. The following are seven causes of prestress loss:

(1) Friction of the prestress steel with the duct and loss due to misalignment of the duct.

(2) Anchorage slip as the strand wedges seat at the bearing plate.

(3) Elastic shortening of the Concrete.

(4) Creep of the concrete.

(5) Shrinkage of the concrete.

(6) Relaxation of the prestress steel.

(7) The stressing Sequence.

Items 3 to 7 above are losses that take effect after stressing is complete and in accordance to Section 50-1-O8 of the Standard Specifications are assumed to be a total of 20 KSI for low relaxation wire and 32 KSI for normal relaxation wire and 22 KSI for bars.

Items 1 and 2 above are losses that occur during the stressing operation and can be calculated knowing the strand properties and the prestress tendon path configuration. These are the losses that are of most concern to the Structure Representative.

FRICTION LOSS

The losses due to friction can be calculated using the following formula:

$$T_0 = T_x e^{(ua)} \qquad \text{Equation 1}$$

where T_o = steel stress at the jacking end.

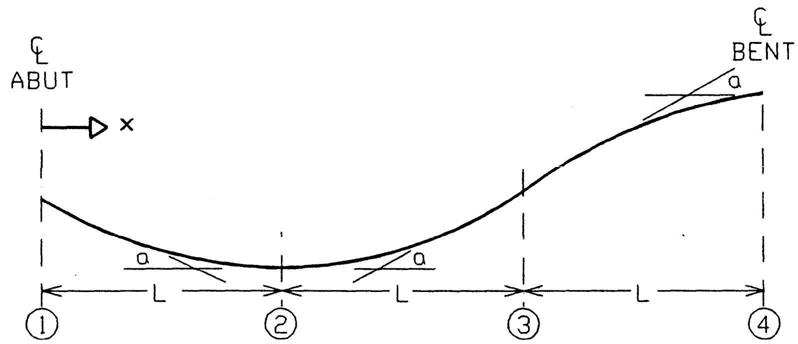
T_x = steel stress at any point x

e = base of Naperian logarithms

u = friction curvature coefficient

a = total angular change of the prestressing steel profile (tendon path) in radians from the jacking end to a point x.

L = length of prestressing steel from the jacking end to a point x.

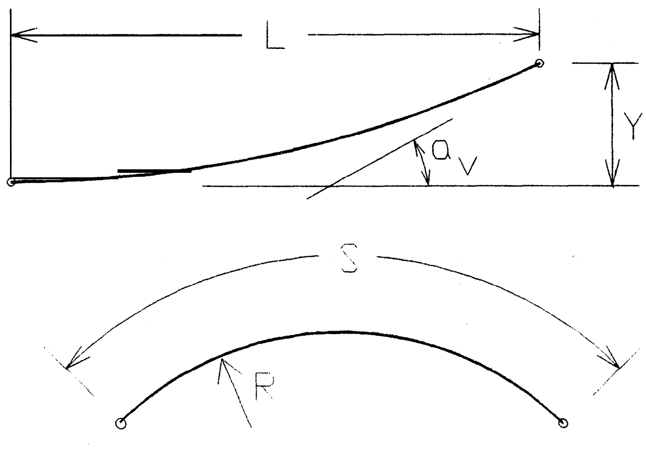


Section 50-l.09 of the Standard Specifications requires that the prestress ducts shall be rigid and galvanized. A friction coefficient of u = 0.20 is given in the Standard Specifications and the Contract Plans.

The prestress steel stress at any point x can be determined by manipulating Eqn 1 as follows:

$$T_x = T_0 e^{-(ua)} \qquad \text{Equation 2}$$

To determine the correction 'a' due to the vertical curvature of the tendon path and for any horizontal bridge curvature that does exist, the following formulas can be used.



$$a_v = 2\frac{Y}{L}$$ Equation 3

$$a_H = \frac{s}{R}$$ Equation 4

$$a = \sqrt{(a_v)^2 + (a_H)^2}$$ Equation 5

Y = tendon drape in length L

S = length of curve from beginning of curve to point x

R = radius

To determine the loss due to friction expressed as a fraction of the temporary jacking stress is

$$\frac{T_o - T_x}{T_o} = 1-e^{-(ua)}$$ Equation 6

The loss that occurs due to the anchor set can be determined using the following approximate formulas:

$$\Delta f = \frac{2dx}{L}$$ Equation 7

$$x = \sqrt{\frac{E(\Delta L)L}{d}} \qquad \text{Equation 8}$$

where Δf = change in stress due to anchor set

d = friction loss in length L

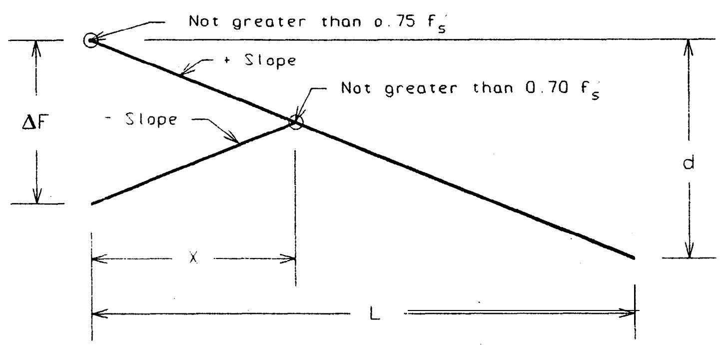
x = length influenced by the anchor set

L = distance to a point where the loss is known

ΔL = anchor set (normally = 3/8")

E = modulus of elasticity, assume 27 x 10 ksi

f_{jack} jacking stress



Section 50-1.08 of the Standard Specifications requires that the maximum temporary stress (jacking stress before anchor set) shall not exceed 75% of the specified minimum ultimate tensile strength of the prestressing steel. In addition, the initial stress shall not exceed 70% of the specified tensile strength of the prestressing steel. This initial stress is just after anchor set but before any long term losses occur, such as concrete shrinkage, relaxation of prestress steel, etc..

ELONGATIONS

As Structure Representative, it will be your responsibility to monitor the contractor's stressing operations. In addition to the use of a load indicator to check prestress force as described earlier in this manual the strand elongations must be measured and compared with the calculated theoretical elongations.

The contractor will submit elongation calculations on the working drawings using assumed values for the modulus of elasticity (E) and the area of the strand (A). When the prestress strand is delivered to the jobsite, it should have a white release tag with the actual E and A written on the back. If these values are not written on the back of this tag, then check the Category 41 file. The E and A should be on the TL-29. The theoretical elongations should be recalculated using the actual E and A.

The elongation between two points where the stress varies linearly can be given by the following equation:

$$\Delta = \frac{T_{avg}\ L}{E} \qquad \text{Equation 9}$$

where T_{avg} = average stress between two points $=(T_1 + T_2)/2$

E = Modulus of Elasticity

L = Length between T_1 and T_2

For almost all field situations the elongations based on the numerical average of the end stresses will yield sufficiently accurate results.

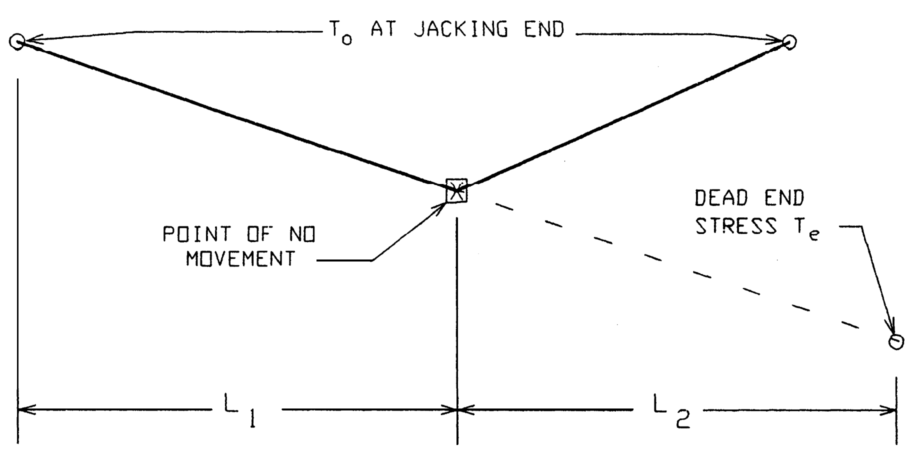
Equation 9 above applies to one-end stressing. For two end simultaneous stressing, the following derivation from Equation 9 can be used.

$$\Delta = \frac{T_0(1+\boxtimes)(L_1 + L_2)}{2E} \qquad \text{Equation 10}$$

where $\boxtimes$ = is the theoretical point of no movement.

The above formulas can be expanded for the entire structure once the theoretical point of no movement or minimum stress is known or calculated. In a continuous structure stressed with two end stressing, the point of no movement in a cable occurs where the losses right of the point equal the losses left of the same point. The force coefficient at that point is shown on the Contract Plans with the symbol, $\boxtimes$.

Reasonably accurate elongation calculations can be made for a structure given the following stress diagram:



If the structure is stressed non-simultaneously, the elongations at the jacking end can be estimated using the assumption that the dead end stress T_e is given by the following formula:

$$T_e = T_0(2\boxtimes - 1) \qquad \text{Equation 11}$$

The first and second elongations are

$$\Delta = \frac{T_0}{2E}\left[(1 + \boxtimes)\,L_1 + (3\boxtimes - 1)\,L_2\right] \qquad \text{Equation 12}$$

and

$$\Delta = \frac{T_0(1 - \boxtimes)L_2}{E} \qquad \text{Equation 13}$$

After obtaining the theoretical elongations, the measurable elongations are calculated. This is usually equal to 80% of the calculated elongation (using the actual E and A) from the first end and 100% from stressing the second end.

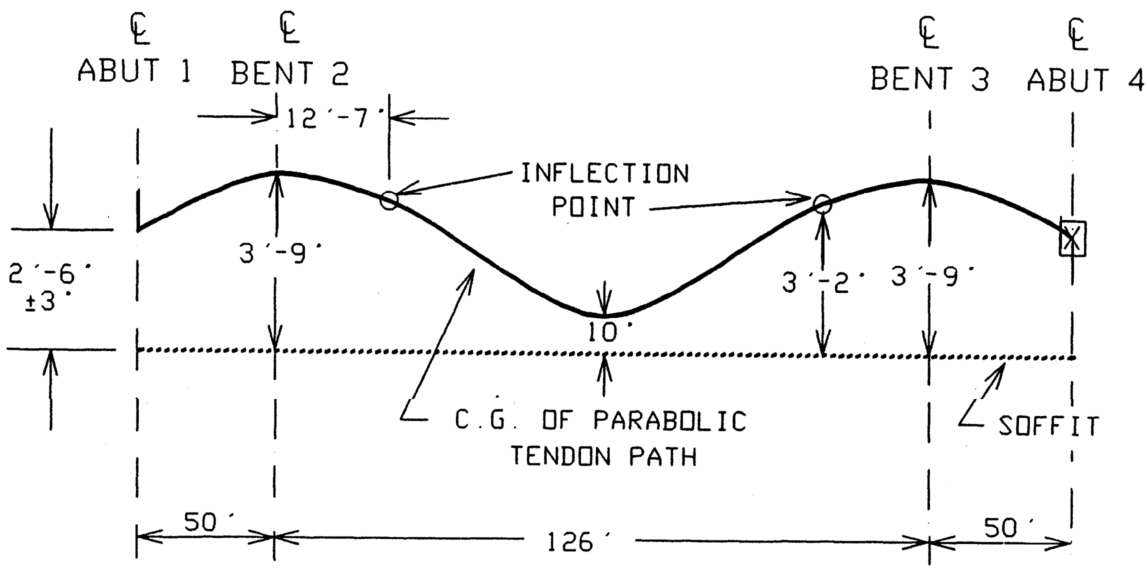
In most cases, the use of the $\boxtimes$ term as shown on the plans will yield acceptable results. Error is introduced because the calculations are based on a straight line stress variation and the term is usually an average of tendons and does not account for tendon path length variations.

EXAMPLE CALCULATIONS

Example l-Continuous Structure Stressed from one end

Given by the Contract Plans:

270 ksi low relaxation strand
P_{jack} = 5150 kips
A = 25.43 in^2
Anchor set = 3/8"
= 0.854 x jacking stress



CG path of the prestressing steel- Figure 1

Given by the Standard Specifications:

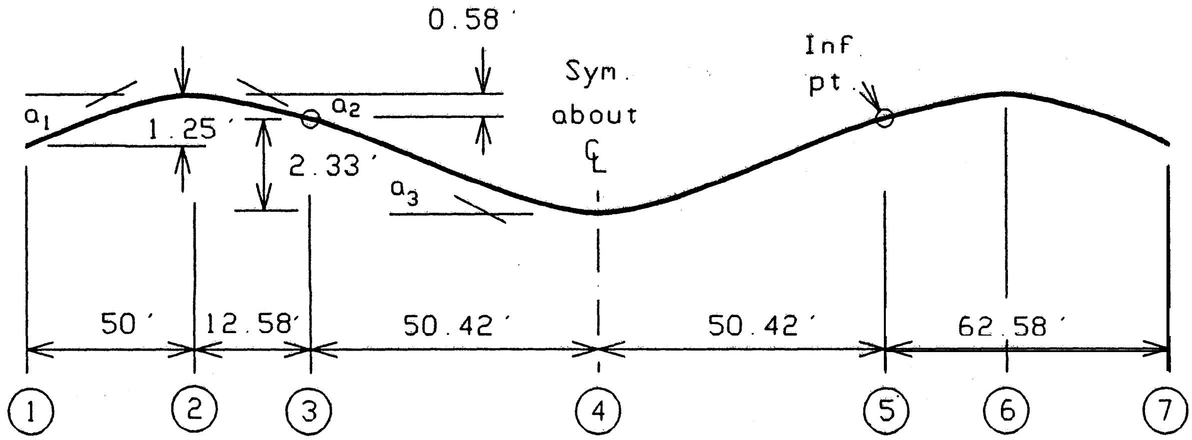
$$T_x = T_0 e^{-(ua)}$$

u = 0.20 for wire strand in rigid galvanized duct.

Losses after stressing for low relaxation steel = 20 ksi.

Find the total elongation?

(1) Total elongation



LOC	a	ua	Σua	$e^{-(ua)}$	x 202.5
1-2	0.050	0.0100	0.0100	0.990	200.5
2-3	0.092	0.0184	0.0284	0.972	196.8
3-4	0.093	0.0186	0.0470	0.954	193.2
4-5	0.093	0.0186	0.0656	0.937	189.7
5-6	0.092	0.0184	0.0840	0.919	186.1
6-7	0.050	0.0100	0.0940	0.910	184.3

Elongation (Δ) = PL/AE; P/A = T_{avg}

$$= \frac{T_{avg} L}{E}$$

Assume E = 27 x 10^3 ksi

Δ = [{(202.5 Ksi + 200.5 Ksi)/2}{50 ft} +
((200.5 + 196.8)/2){12.58} +
{(196.8 + 193.2)/2}{50.42} +
{(193.2 + 189.7)/2}{50.42} +
{(189.7 + 186.1)/2}{12.58} +
{(186.X + 184.3)/2}{50.00}](12 in/ft) / 27 x 10^3 Ksi

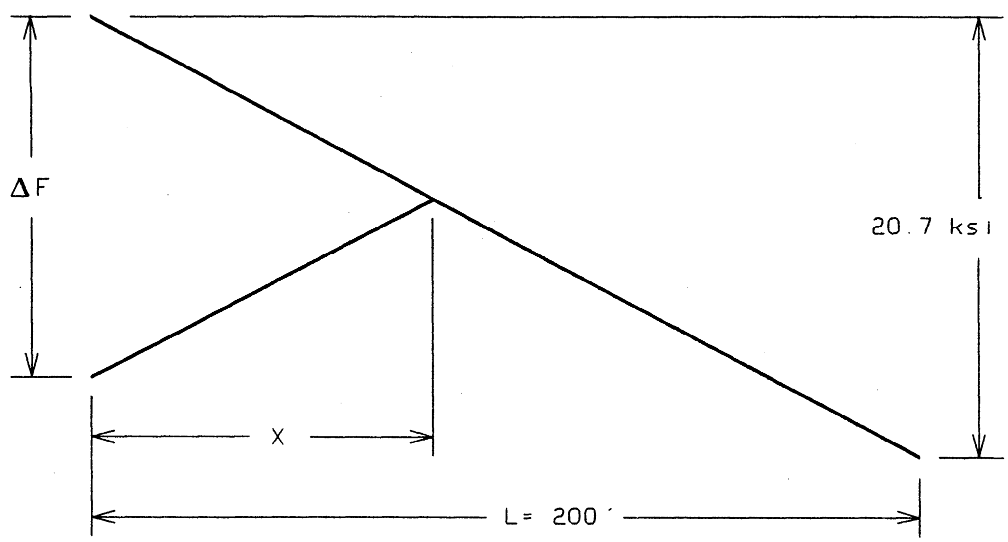
Δ = 19.4 in.

Example 2 - Anchor set calculation

The contract plans require a 3/8" anchor set. What is the change in stress at the anchorage end (jacking end) and how far into the structure does the anchor set loss extend?

Given: $E = 27 \times 10^3$ Ksi and ΔL = 3/8"

Friction loss in length L = 20.7 Ksi



$$X = \sqrt{\frac{E(\Delta L)L}{d}} = \sqrt{\frac{(27 \times 10^3\ \text{Ksi})(0.375\ \text{in})(200\ \text{ft})}{(20.7\ \text{Ksi})(12\ \text{in/ft})}}$$

$$= 90.29\ \text{ft}$$

$$\Delta f = \frac{2dx}{L} = \frac{(2)(20.7\ \text{Ksi})(90.29\ \text{ft})}{200\ \text{ft}}$$

$$= 18.69\ \text{Ksi}$$

Example 3 - Simple span stressed at one end

Given by the Contract Plans:

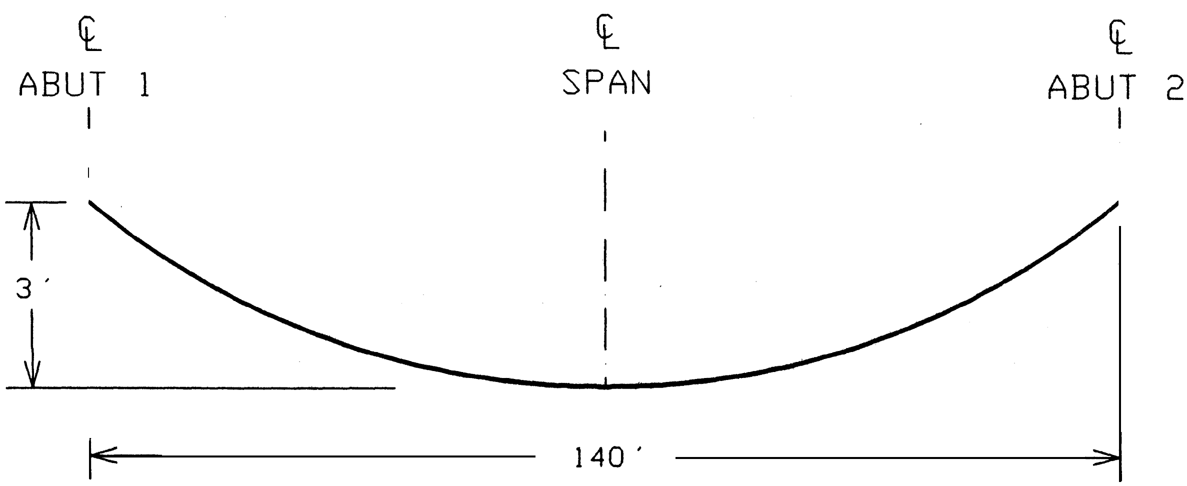
270 Ksi Low Relaxation Strand

P_{jack} = 12575 Kips

Area (A) of prestressing steel = 64.88 in^2

Anchor set = 3/8"

One end stressing



CG of the prestressing path

Find:

1. Is the jacking stress less than or equal to 0.75 f'_s ?
2. Is the intial stress after the anchor set less than or equal to 0.70 f'_s ?
3. Find the final working force at the centerline of span.
4. Find the theoretical elongation.

Example 3 (cont.)

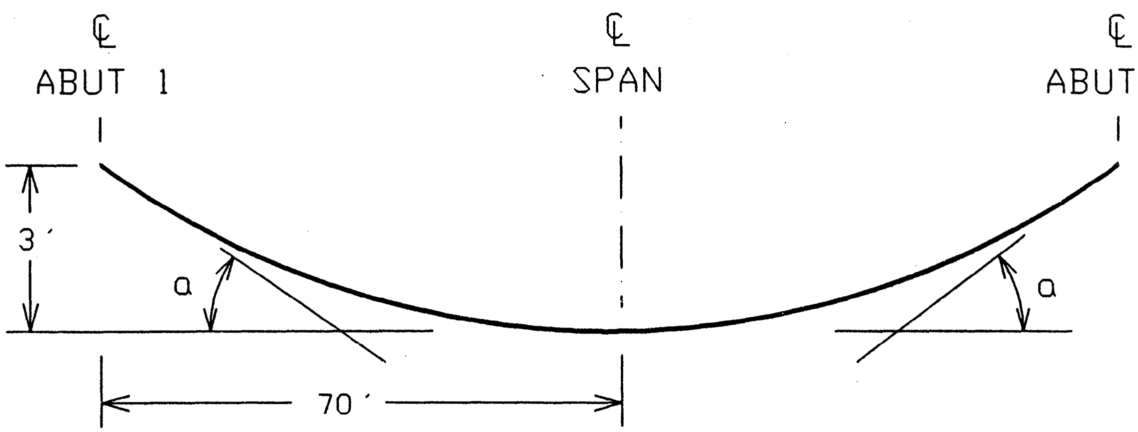
1. Jacking stress

$$\frac{P_{jack}}{\text{Area(A)}} = \frac{12575 \text{ Kips}}{64.88 \text{ in}^2}$$

$$= 193.8 \text{ Ksi}$$

0.75(270 Ksi) = 202.5 Ksi OK

2. Intial stress at the dead end (abutment 2)



$$a = \frac{(2)(3)}{70} = 0.0857 \times 2 = 0.1714$$

$$T_x = T_o e^{-(ua)} = (193.8) e^{-[(0.20)(0.1714)]}$$

$$T_x = (193.8)(0.9663) = 187.3 \text{ Ksi}$$

Example 3 (cont.)

Effect of the anchor set.

$$X = \sqrt{\frac{E(\Delta L)L}{d}} = \sqrt{\frac{(27 \times 10^3 \text{ Ksi})(0.375 \text{ in})(140 \text{ ft})}{(193.8 - 187.3 \text{ Ksi})(12 \text{ in/ft})}}$$

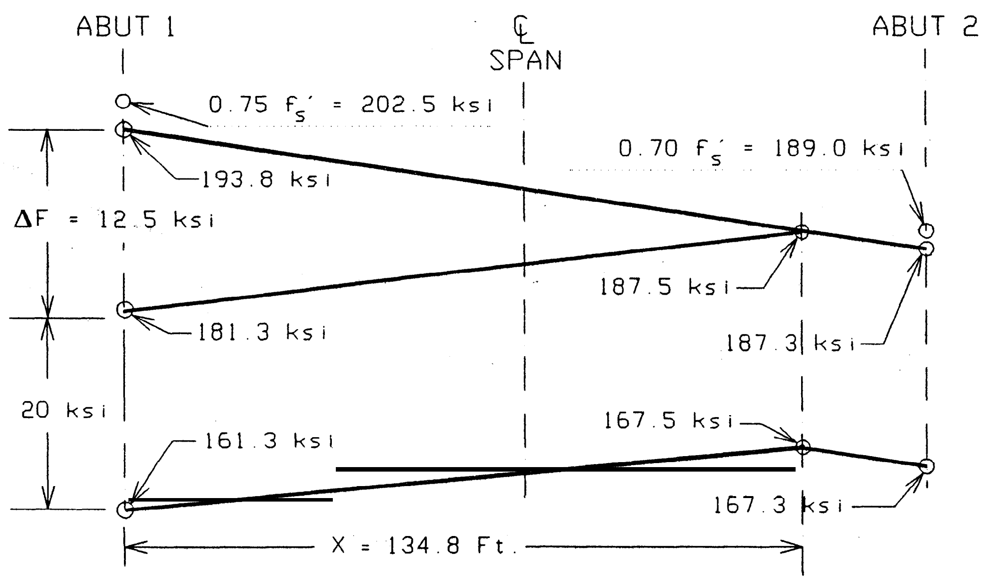
$$= 134.5 \text{ ft}$$

$$\Delta F = \frac{2dx}{L} = \frac{(2)(193.8 - 187.3 \text{ Ksi})(134.5 \text{ ft})}{140 \text{ ft}}$$

$$= 12.5 \text{ Ksi}$$

Intial stress after anchor set.

Stress Diagram



Initial stress = 187.5 Ksi < 0.70 f'_s = 189.0 Ksi

3. Final working force at the centerline of span.

(164.3 Ksi)(64.88 in^2) = 10,660 Kips

4. Theoritical elongation

$$= \frac{T_{avg} L}{E} = \frac{[(193.8 + 187.3)/2 \text{ Ksi}](140 \text{ ft})(12 \text{ in/ft})}{27 \times 10^3 \text{ Ksi}}$$

$$= 11.9 \text{ inches}$$